~~Intersecting Brane Worlds~~ Cobordism on K3 and CY manifolds

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(RB, Cribiori, arXiv:2112.07678)

(RB, Cribiori, Kneißl, Makridou, arXiv:2205.09782+arXiv:2207.nnnnn)

Introduction

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Conjecture: No global symmetries in QG!

- If one seems to detect one, it actually needs to be gauged or broken
- For continuous symmetries, this means:

$$d \star F_{d-n+1} = \star J_{d-n}\,, \quad \text{or} \quad d \star J_{d-n} = I_{n+1}$$

- Non-vanishing cobordism groups $\Omega_n^{\widetilde{QG}} \neq 0$ are a source of global symmetry and thus need to be nullified eventually, i.e $\Omega_n^{QG} = 0$ (McNamara,Vafa, 1909.10355).
- Discussed many examples based on $\Omega_n^{\rm Spin}$, $\Omega_n^{{\rm Spin}^c}$ and which defects can break the global symmetries.(see also (Dierigl,Heckmann, 2012.00013))

Introduction

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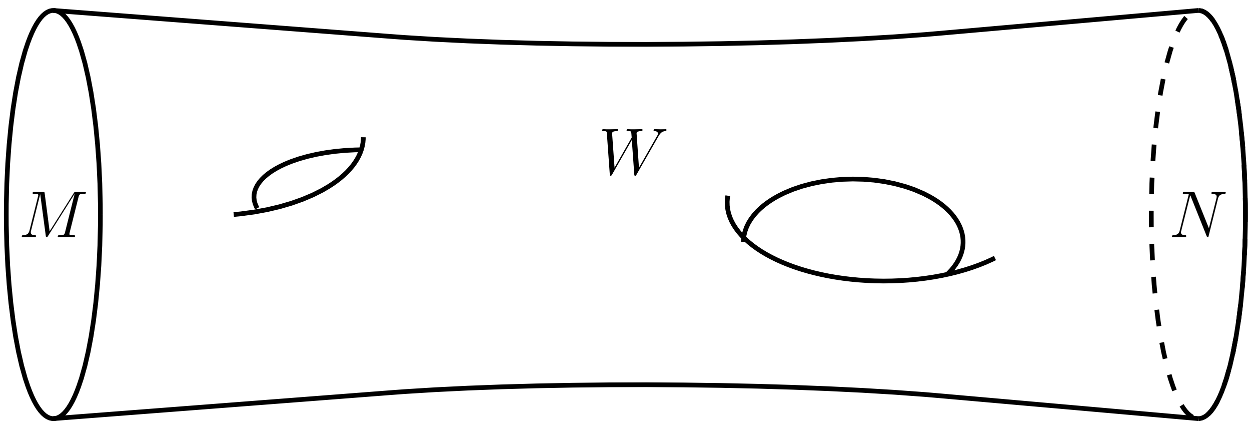
Spin-cobordism Ω_n^{Spin} are equivalence classes of n-dim. Spin-manifolds, where M and N are equivalent if

$$\partial W = M \sqcup \overline{N}\,.$$



The addition is defined via disjoint union

$$[M] + [N] = [M \sqcup N]\,.$$

Cobordism groups also appeared in Dai-Freed anomalies

(Etxebarria,Montero, 1808.00009)

Introduction

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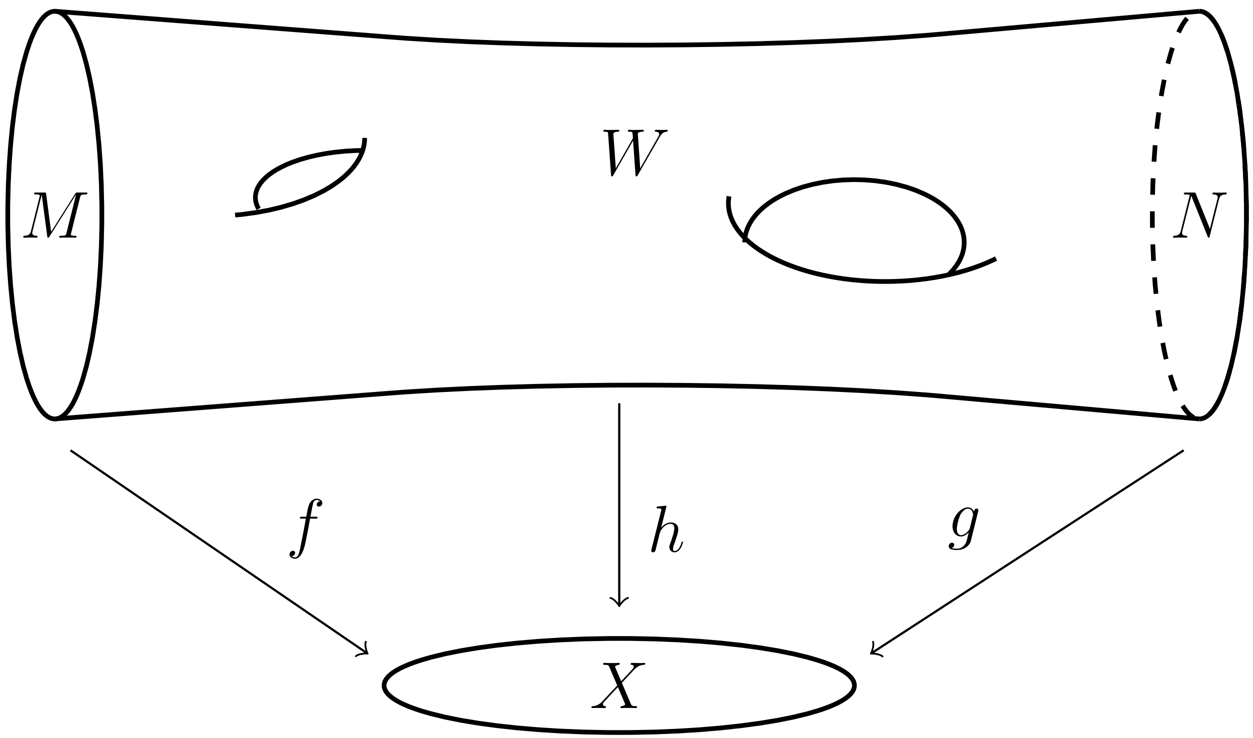
Generalization to $\Omega_n^G(X)$: cobordism groups relative to X consisting of pairs (M, f) modulo equivalence:



$X = pt$ is the former case.

Introduction

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Ω_n^{Spin} and their generators: obvious relation to $\widetilde{KO}(S^n)$

n	0	1	2	3	4	5	6	7	8
Ω_n^{Spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}^2$
$\Sigma_{n,i}$	pt$^+$	S^1_p	$S^1_p \times S^1_p$	0	$K3$	0	0	0	$\mathbb{B} \oplus \mathbb{HP}^2$

$\Omega_n^{\text{Spin}^c}$: Obvious relation to $\widetilde{K}(S^n)$

n	0	2	4	6
$\Omega_n^{\text{Spin}^c}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^2$
$\Sigma_{n,i}$	pt$^+$	$\mathbb{P}^1$	$\mathbb{P}^2 \oplus (\mathbb{P}^1)^2$	-
Inv.	1	c_1	td, c_1^2	td, $c_1^3/2$

K-theory and Cobordism

K-theory and Cobordism

- There exist an intricate mathematical relationship between K-theory and cobordism.
- What is the physical significance of it?

Atiyah–Bott–Shapiro (ABS): There exist ring homomorphisms

$$\alpha^c : \Omega_*^{\mathrm{Spin}^c}(pt) \to K_*(pt)\,, \qquad \alpha : \Omega_*^{\mathrm{Spin}} \to KO_*(pt)\,.$$

with $K_n(pt) = \tilde{K}(S^n)$. When restricted to a fixed grade n

$$\alpha_n^c([M]) = \mathrm{Td}(M) \in \mathbb{Z}\,.$$

The map α^c is a cobordism invarians and surjective, so that one can divide by its kernel to get an isomorphism

$$\Omega_n^{\mathrm{Spin}^c}/\ker(\alpha) \cong \widetilde{K}(S^n)$$

ABS map

ABS map

An explicit definition of the map α is

$$\alpha_n([M]) = \begin{cases} \hat{A}(M) & n = 8m, \\ \hat{A}(M)/2 & n = 8m+4, \\ \dim H \mod 2 & n = 8m+1, \\ \dim H^+ \mod 2 & n = 8m+2, \\ 0 & \text{otherwise}, \end{cases}$$

This has been generalised to the Hopkins–Hovey isomorphism

$$\Omega_*^{\mathrm{Spin}}(X) \otimes_{\Omega_*^{\mathrm{Spin}}} KO_* \to KO_*(X),$$
$$\Omega_*^{\mathrm{Spin}^c}(X) \otimes_{\Omega_*^{\mathrm{Spin}^c}} K_* \to K_*(X)$$

for any background topological space X ($X = pt$ is the former case).

Gauging global symmetries

Gauging global symmetries

- Both K-theory and cobordism compute global charges
- K-theory global symmetries are all expected to be gauged (Freed, hep-th/0011220)
- For non-torsion classes ($K_n(pt) = \mathbb{Z}$) this leads to Bianchi identities of the form (for D_{9-n} branes)

$$d\tilde{F}_{n-1} = \sum_i N_i \delta^{(n)}(\Sigma_i) + \ldots$$

- Proposal: the missing "geometric" piece is described by the corresponding cobordism group $\Omega_n^{\mathrm{Spin}^c}(pt)$

In string theory:

- $\Omega_n^{\mathrm{Spin}}(pt)/\Omega_n^{\mathrm{Spin}^c}(pt)$ describes the geometric contribution in type I/type IIB orientifold(F-theory) tadpole constraints

Example: Spin^c

Example: Spin^c

- All K-theory classes $K_{2n}(pt) = \mathbb{Z}$ are gauged
- The Todd classes are the natural currents

$$\star J_{d-n} = \alpha^c([M]) = \mathrm{td}_n(M),$$

- Note, there appears a proliferance of $\mathbb{Z}$ factors in higher cobordism classes $\Omega_n^{\mathrm{Spin}^c}(pt)$.

For $\Omega_2^{\mathrm{Spin}^c}$ one has $\alpha_2^c(M) = \mathrm{td}_2(M) = c_1(M)/2$, leading to the F-theory/type IIB orientifold relation

$$d\tilde{F}_1 = \sum_i N_i\, \delta^{(2)}(\Delta_{8,i}) - 24\, \alpha_2^c(M)\,.$$

$M = \mathbb{P}^1$: downstairs geometry of $T^2/\Omega I_2(-1)^{F_L}$.

Example: Spin^c

Example: Spin^c

For $\Omega_6^{\mathrm{Spin}^c} = \mathbb{Z} \oplus \mathbb{Z}$, the gauging leads to a D3-brane tadpole

$$d\tilde{F}_3 = \sum_i N_i\, \delta^{(6)}(\Delta_{4,i}) + a_1^{(6)} \frac{c_2(M)c_1(M)}{24} + a_2^{(6)} \frac{c_1^3(M)}{2}\,.$$

For $a_1^{(6)} = -12$, $a_2^{(6)} = -30$ and $M = B_3$ the base of a smooth elliptically fibered CY fourfold Y, this is the $D3$ tadpole of F-theory

$$\chi(Y)/24 = \int_B \left(\frac{1}{2}\, c_2(B)\, c_1(B) + 15\, c_1^3(B) \right)\,.$$

- contains precisely the two cobordism invariants td_6 and $c_1^3/2$, i.e. in particular no c_3-term.
- appearance of $c_1^3/2$ is related to the presence of $O7$-planes. Pure $O3$-planes: $M = dP_9 \times \mathbb{P}^1$

Gauging of Ω_1^{Spin}?

Gauging of Ω_1^{Spin}?

Hopkins-Hovey isomorphisms: $\widetilde{KO}(S^1) \simeq \Omega_1^{\mathrm{Spin}} = \mathbb{Z}_2$

Gauging: $\mathbb{Z}_2$ valued charge neutrality condition

$$\int_M \sum_i N_i \, \delta^{(1)}(\Delta_{9,i}) = K \, \alpha_1(M) \qquad \mathrm{mod}\ 2\,.$$

What is the value of K?

- K even: r.h.s. decouples $\Rightarrow$ K-theory charge gauged + charge $\Omega_1^{\mathrm{Spin}} = \mathbb{Z}_2$ needs to be broken by 7-defects
- K odd: a single non-BPS $\widehat{D8}$-brane on the background $M = S_p^1$ would be charge neutral and allowed, unexpected $\Rightarrow$ K even

Generic situation for torsion charges?

Hopkins–Hovey isomorphism

Hopkins–Hovey isomorphism

Recall the Hopkins–Hovey isomorphism (generalization of a classic theorem by Conner-Floyd)

$$\Omega_*^{\mathrm{Spin}}(X) \otimes_{\Omega_*^{\mathrm{Spin}}} KO_* \to KO_*(X),$$

$$\Omega_*^{\mathrm{Spin^c}}(X) \otimes_{\Omega_*^{\mathrm{Spin^c}}} K_* \to K_*(X)$$

are isomorphisms for any topological space X.

It involves the more refined generalized cobordism $\Omega_n^{\mathrm{Spin}^c}(X)$ and K-theory $K_n(X)$ classes.

- How to compute them?
- What is their physical relevance in the swampland program?

Upcoming work: (Bhg,Cribiori, Kneissl, Makridou, 2207.nnnnn)

Generalized cobordism

Generalized cobordism

Clearly, this introduces some background dependence

$$\Omega_n^{\mathrm{Spin}^c}(X) \to \Omega_n^{\mathrm{Spin}^c}(pt) \to \Omega_n^{\mathrm{QG}} = 0$$

Physical expectation for $K^{-n}(X)$ with $k = \dim(X)$

- Classifies all D-brane charges in $D = 10 - k$ dimensions
- Contributions from wrapped 10D branes subject to Freed-Witten anomalies
- New tachyon decay channels can exist

Mathematically: compute $K^{-n}(X)$ via the Atiyah-Hirzebruch spectral sequence (AHSS)

- FW anomalies: non-trivial maps $d_r : E_r^{p,q} \to E_r^{p+r,q-r+1}$
- New decay channels: extension problem at the end of AHSS, $e(\mathbb{Z}_2, \mathbb{Z}) = \{\mathbb{Z} \oplus \mathbb{Z}_2, \mathbb{Z}\}$

Generalized cobordism

Generalized cobordism

Approach: For physical interpretation, study simple cases

For $X \in \{S^k, T^k, K3, CY_3^{(\pi^1=0)}\}$ we find the simple result

$$K^{-n}(X) = \bigoplus_{m=0}^{k} b_{k-m}(X) \cdot \underbrace{K^{-n-m}(pt)}_{\text{10D branes}} \,.$$

Consistent with dimensional reduction

$\Omega_n^{\mathrm{Spin}^c}(X)$ are also computed via an AHSS

$$\Omega_{n+k}^{\mathrm{Spin}^c}(X) = \bigoplus_{m=0}^{k} b_m(X) \cdot \Omega_{n+k-m}^{\mathrm{Spin}^c}(pt)$$

It follows the same pattern! (interesting story for $-k \leq n < 0$).

Generalized cobordism

Generalized cobordism

Therefore, the ABS orientation can be extended to a map

$$\alpha^c_X : \Omega^{\mathrm{Spin}^c}_{n+k}(X) \to K_{n+k}(X)$$

that acts like α^c on each term $\Omega^{\mathrm{Spin}^c}_{n+k-m}(pt)$.

Corrolar: gauging of global symmetries, tadpoles also follow the rules of dimensional reduction

The HH isomorphism says that this will also hold in general!

(more details in talk by Christian Kneißl)

Dynamical Cobordism

Dynamical Cobordism

Breaking of $\Omega_1^{\text{Spin}} = \mathbb{Z}_2$: approach this question via dynamical cobordism (Bhg,Cribiori, Kneissl, Makridou, 2205.09782)

New example for dynamical cobordism (Buratti,Delgado,Uranga, 2104.02091),(Buratti,Calderón-Infante,Delgado,Uranga, 2107.09098).

Consider the backreaction of a neutral, positive tension domain wall in dilaton-gravity.

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}\left(R - \frac{1}{2}(\partial\Phi)^2\right) - T\int d^{10}x\sqrt{-g}\, e^{\frac{5}{4}\Phi}\,\delta(r)$$

- Solution $S^1 \times I_1$ with one non-trivial longitudinal direction, preserving 8D Poincare symmetry (Bhg,Font, hep-th/0011269)
- Issue: extra log-singularities $\rightarrow$ ETW 7-branes

Dynamical Cobordism

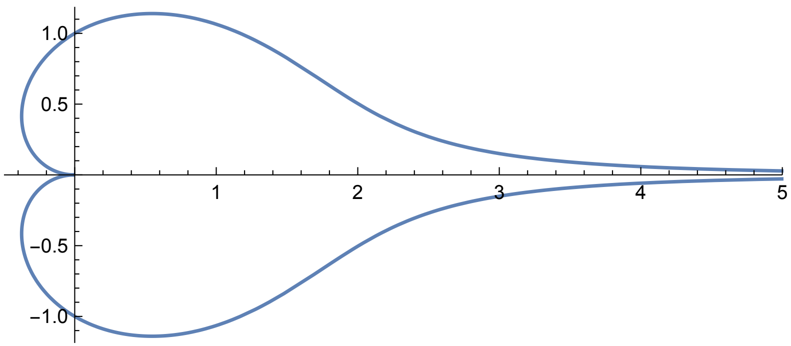
Dynamical Cobordism

What is the nature of the ETW 7-brane?

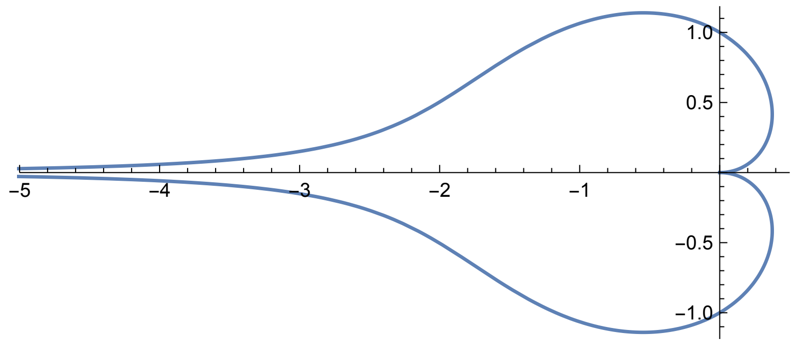
Constraints:

- preserves 8D Poincare symmetry
- has $\log \rho$ singularities close to its core
- is non-isotropic in the two transverse directions.

ETW 7^- brane



ETW 7^+ brane



Source: $S_7 = -T_7 \int d^{10}x \sqrt{-g}\, \frac{\delta(\rho)}{2\pi\rho}$ with $\kappa_{10}^2 T_7 = 2\pi$.

(more details in talk by Andriana Makridou)

Conclusions

Conclusions

The HH-isomorphism of K-theory and cobordism classes provides the mathematical framework for the generic gauging of the global symmetries involved

- K-theory classes = cobordism classes of D-defects.
- Gauging leads to tadpole cancellation conditions known from orientifolds and F-theory.
- K-theory provides the brane charges, cobordism the geometric contributions to tadpoles

There are still open questions

- Generalization to type IIA?
- Explicit computation of $\Omega_n^{\mathrm{Spin+def};U(1)_p}$ classes?
- Unique bottom-up result for the final $\Omega_n^{QG} = 0$?

  (Andriot,Carqueville,Cribiori, 2204.00021)